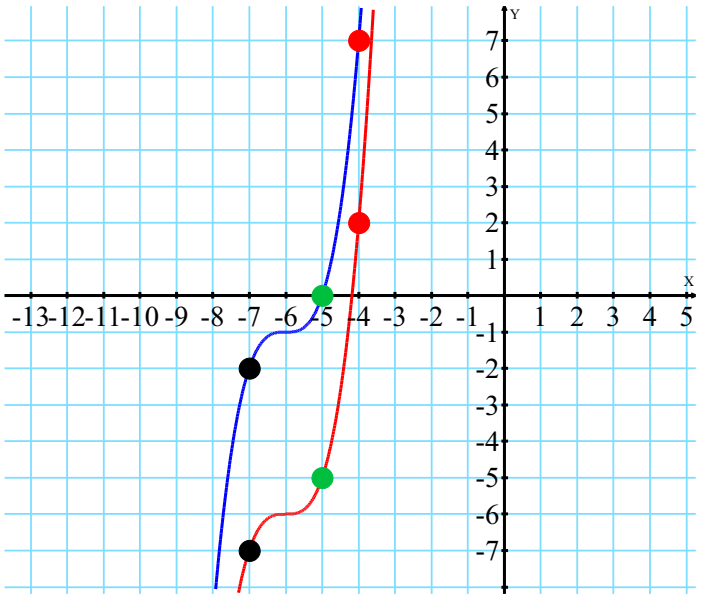
1.

Vertical Shift Up 5 Units of $y = (x + 6)^3 - 6$
To Get $y = (x + 6)^3 - 1$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(x, y + 5)
(-7, -7)	(-7, -2)
(-5, -5)	(-5, 0)
(-4, 2)	(-4, 7)

Vertical Shift Up 5 Units

Add 5 to y.

$y = (x + 6)^3 - 6$

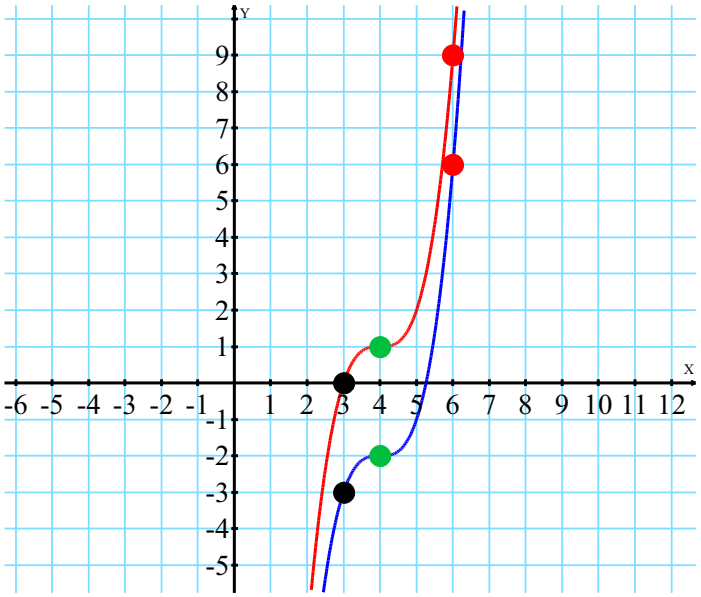
$y = [(x + 6)^3 - 6] + 5$

$y = (x + 6)^3 - 6 + 5$

$y = (x + 6)^3 - 1$

2.

Vertical Shift Down 3 Units of $y = (x - 4)^3 + 1$
To Get $y = (x - 4)^3 - 2$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(x, y - 3)
(3, 0)	(3, -3)
(4, 1)	(4, -2)
(6, 9)	(6, 6)

Vertical Shift Down 3 Units

Subtract 3 from y.

$y = (x - 4)^3 + 1$

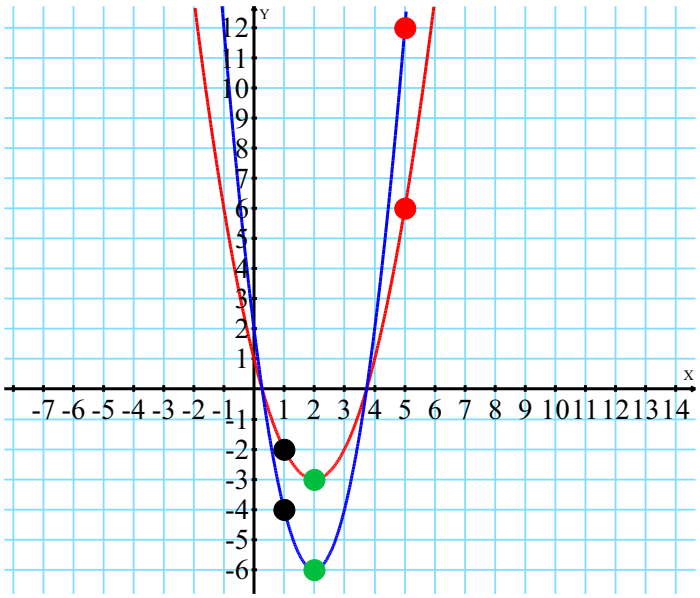
$y = [(x - 4)^3 + 1] - 3$

$y = (x - 4)^3 + 1 - 3$

$y = (x - 4)^3 - 2$

3.

Vertical Stretch (Vertical Expansion) By a Factor of 2 of $y = (x + 2)^2 - 3$
To Get $y = 2(x + 2)^2 - 6$

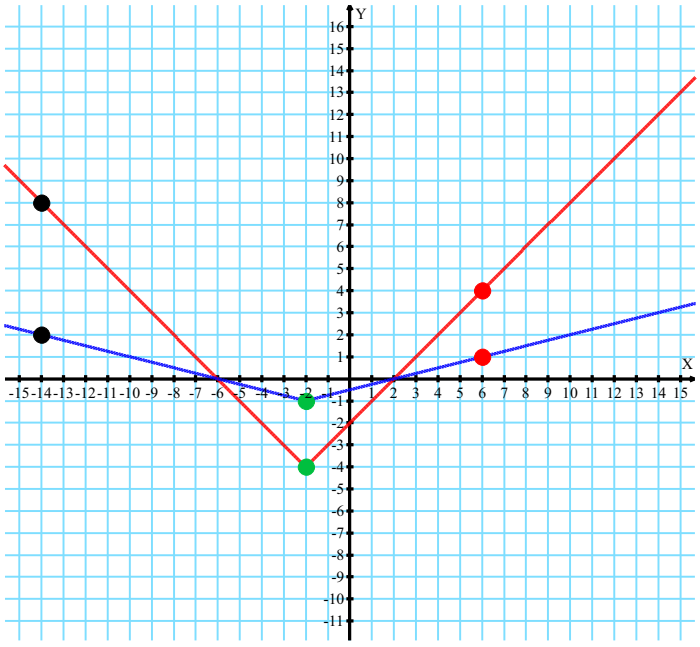


Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x , 2 y)
(1, -2)	(1, -4)
(2, -3)	(2, -6)
(5, 6)	(5, 12)

Vertical Stretch by a Factor of 2
Multiply y by 2
$y = (x + 2)^2 - 3$
$y = 2[(x + 2)^2 - 3]$
$y = 2(x + 2)^2 - 6$

4.

Vertical Shrink (Vertical Compression) By a Factor of 1/4 of $y = (x + 2)^2 - 4$
To Get $y = (1/4)(x + 2)^2 - 1$



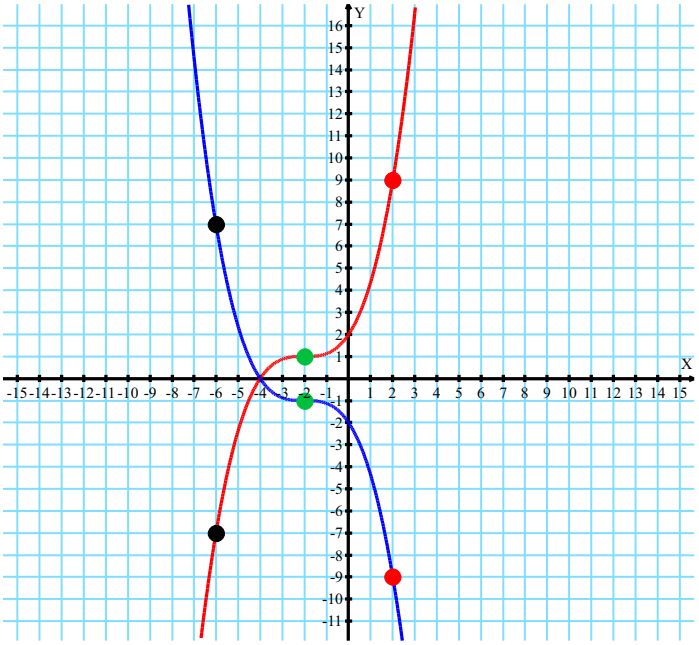
Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x, (1/4) y)
(-14, 8)	(-14, 2)
(-2, -4)	(-2, -1)
(6, 4)	(6, 1)

Horizontal Shrink By a Factor of 1/4
Multiply y by 1/4
$y = (x + 2)^2 - 4$
$y = (1/4)[(x + 2)^2 - 4]$
$y = (1/4)(x + 2)^2 - 1$

5.

Vertical Reflection (Reflect Across x-axis) of $y = (.5x + 1)^3 + 1$

To Get $y = -(.5x + 1)^3 - 1$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(x , -1 y)
(-6, -7)	(-6, 7)
(-2, 1)	(-2, -1)
(2, 9)	(2, -9)

Reflect | Reflect

Vertical Reflection

multiply y by -1

$y = (.5x + 1)^3 + 1$

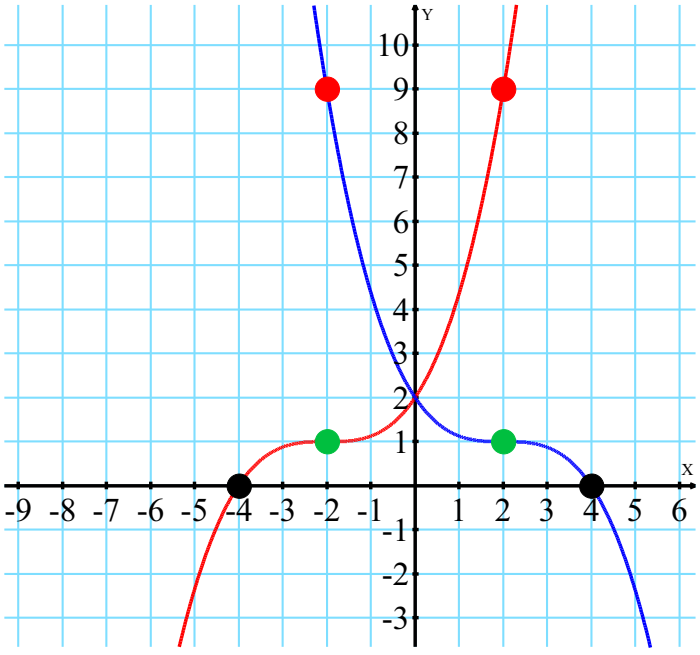
$y = -1[(.5x + 1)^3 + 1]$

$y = -(.5x + 1)^3 - 1$

6.

Horizontal Reflection (Reflect Across y-axis) of $y = ((1/2)x + 1)^3 + 1$

To Get $y = (-(1/2)x + 1)^3 + 1$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(-1 x , y)
(-4, 0)	(4, 0)
(-2, 1)	(2, 1)
(2, 9)	(-2, 9)

Reflect | Reflect

Horizontal Reflection

multiply x by -1

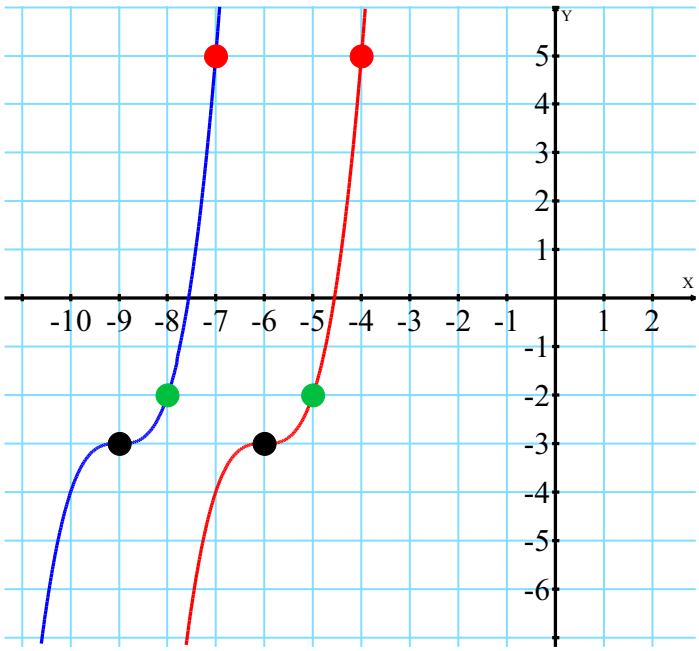
$y = ((1/2)x + 1)^3 + 1$

$y = (-1(1/2)x + 1)^3 + 1$

$y = (-(1/2)x + 1)^3 + 1$

7.

Horizontal Shift Left 3 Units of $y = (x + 6)^3 - 3$
To Get $y = (x + 9)^3 - 3$



Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x - 3, y)
(-6, -3)	(-9, -3)
(-5, -2)	(-8, -2)
(-4, 5)	(-7, 5)

Horizontal Shift Left 3 Units ←

Add 3 to x.

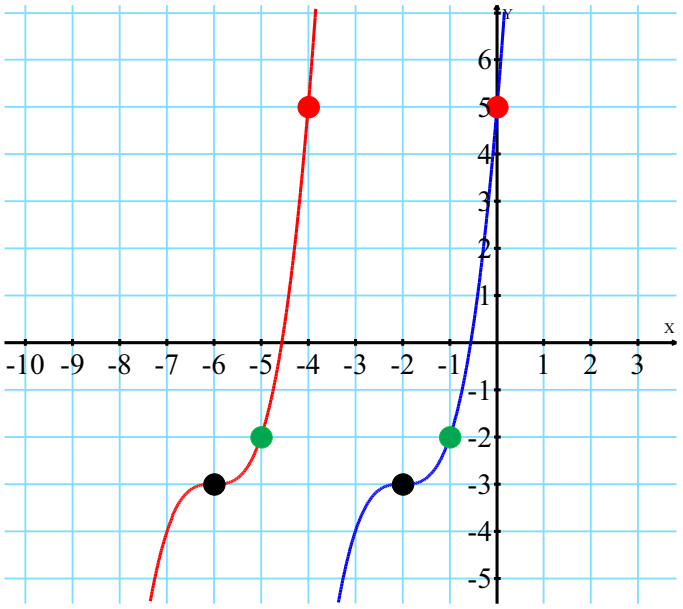
$y = (x + 6)^3 - 3$

$y = (x + 3 + 6)^3 - 3$

$y = (x + 9)^3 - 3$

8.

Horizontal Shift Right 4 Units of $y = (x + 6)^3 - 3$
To Get $y = (x + 2)^3 - 3$



Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x + 4, y)
(-6, -3)	(-2, -3)
(-5, -2)	(-1, -2)
(-4, 5)	(0, 5)

Horizontal Shift Right 4 Units →

Subtract 4 from x.

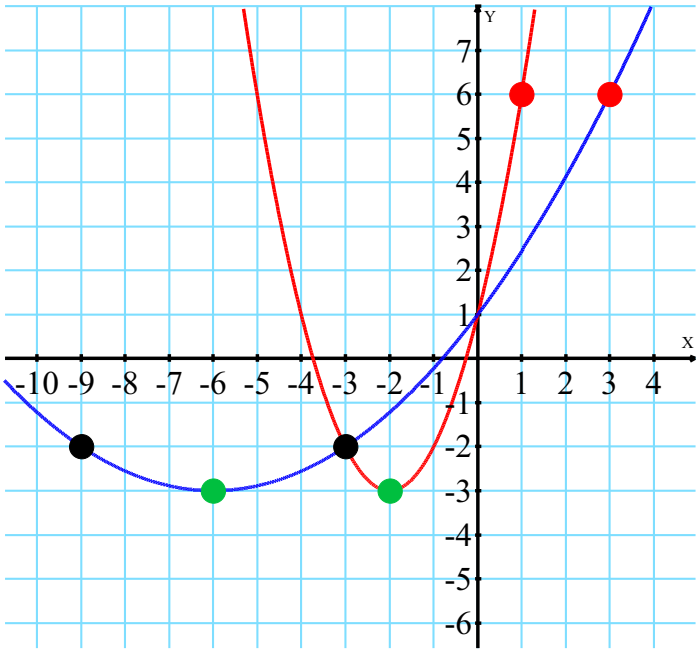
$y = (x + 6)^3 - 3$

$y = (x - 4 + 6)^3 - 3$

$y = (x + 2)^3 - 3$

9.

Horizontal Stretch (Horizontal Expansion) By a Factor of 3 of $y = (\quad x + 2)^2 - 3$
To Get $y = ((1/3)x + 2)^2 - 3$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	(3 x , y)
(-3, -2)		(-9, -2)
(-2, -3)		(-6, -3)
(1, 6)		(3, 6)

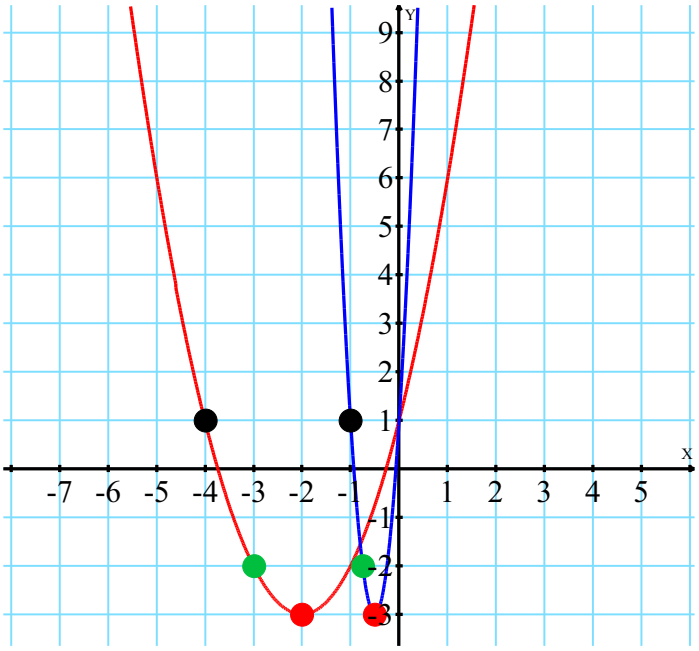
Horizontal Stretch by a Factor of 3

Multiply x by the reciprocal of 3 which is 1/3.

$y = (\quad x + 2)^2 - 3$
$y = ((1/3)x + 2)^2 - 3$
$y = ((1/3)x + 2)^2 - 3$

10.

Horizontal Shrink (Horizontal Compression) By a Factor of 1/4 of $y = (x + 2)^2 - 3$
To Get $y = (4x + 2)^2 - 3$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	((1/4) x, y)
(-4, 1)		(-1, 1)
(-3, -2)		(-3/4, -2)
(-2, -3)		(-1/2, -3)

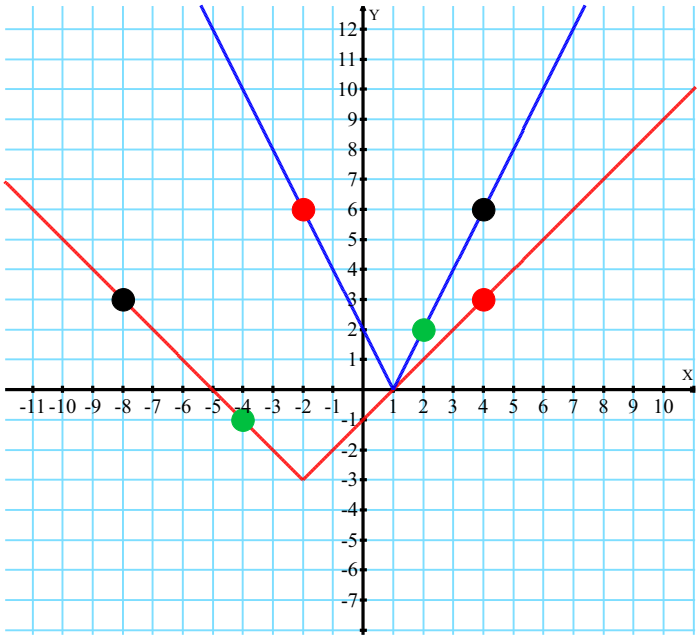
Horizontal Shrink By a Factor of 1/4

Multiply the x by the reciprocal of 1/4 which is 4.

$y = (\quad x + 2)^2 - 3$
$y = (4x + 2)^2 - 3$
$y = (4x + 2)^2 - 3$

11.

Horizontal Compression by 1/2, Horizontal Reflection, Vertical Shift Up 3 Units of $y = |x + 2| - 3$ To Get $y = |-2x + 2|$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	((-1/2) x, y + 3)
(-8, 3)	(4, 6)
(-4, -1)	(2, 2)
(4, 3)	(-2, 6)

→ || ← Reflect | Reflect ↑

Horizontal Compression by 1/2, Horizontal Reflection, Vertical Shift Up 3 Units

Multiply x by the reciprocal of -1/2 which is -2, then add 3 to y.

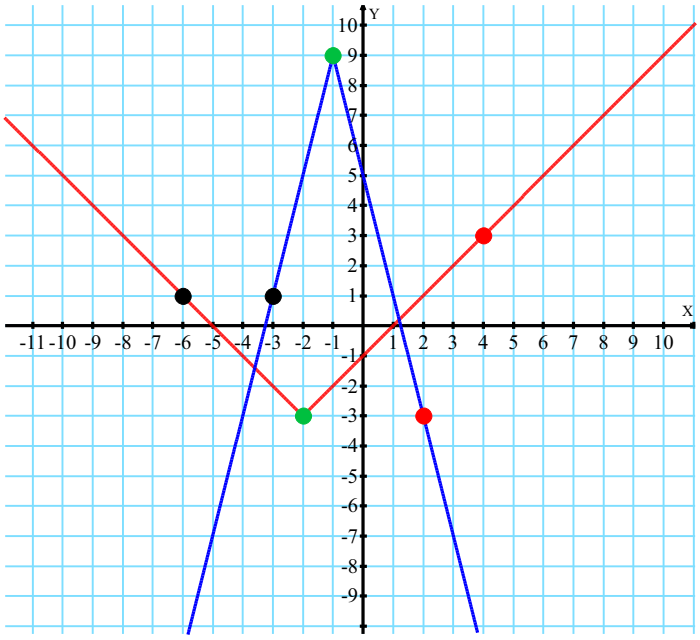
$y = |x + 2| - 3$

$y = |-2x + 2| - 3 + 3$

$y = |-2x + 2|$

12.

Horizontal Compression by 1/2, Vertical Stretch by 2, Vertical Reflection, Vertical Shift Up 3 Units of $y = |x + 2| - 3$ To Get $y = -2|2x + 2| + 9$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	((1/2) x, -2y + 3)
(-6, 1)	(-3, 1)
(-2, -3)	(-1, 9)
(4, 3)	(2, -3)

→ || ← Reflect ↑ Reflect

Horizontal Compression by 1/2, Vertical Stretch by 2, Vertical Reflection, Vertical Shift Up 3 Units

Multiply x by the reciprocal of 1/2 which is 2, then multiply y by -2, and then add 3 to y

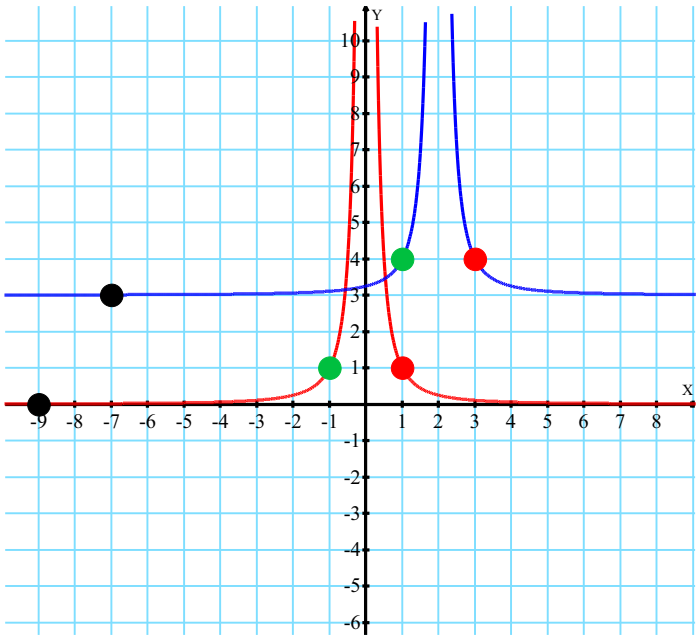
$y = |x + 2| - 3$

$y = -2[|2x + 2| - 3] + 3$

$y = -2|2x + 2| + 9$

13.

Horizontal Shift Right 2 Units, Vertical Shift Up 3 Units of $y = 1/x^2$
To Get $y = 1/(x - 2)^2 + 3$



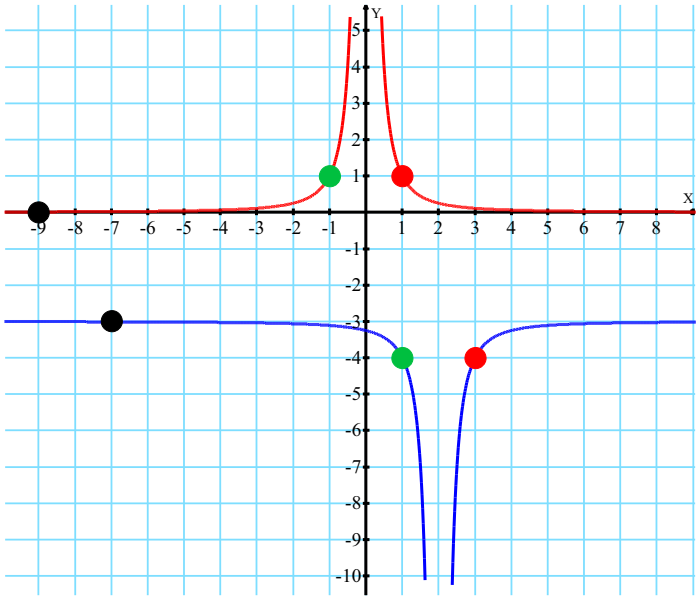
Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x + 2, y + 3)
(-9, 1/81)	(-7, 3 + 1/81)
(-1,1)	(1, 4)
(1, 1)	(3, 4)

Horizontal Shift Right 2 Units,
Vertical Shift Up 3 Units
Subtract 2 from x, add 3 to y.

$y = 1/x^2$
$y = [1/(x - 2)^2] + 3$
$y = 1/(x - 2)^2 + 3$

14.

Horizontal Shift Right 2 Units, Vertical Shift Up 3 Units, Then Reflect over the x-axis
of $y = 1/x^2$ To Get $y = -1/(x - 2)^2 - 3$



Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(x + 2, -(y + 3))
(-9, 1/81)	(-7, -(3 + 1/81))
(-1,1)	(1, -4)
(1, 1)	(3, -4)

Horizontal Shift Right 2 Units,
Vertical Shift Up 3 Units, then
Reflect Over The X-Axis
Subtract 2 from x, add 3 to y,
then multiply y by -1.

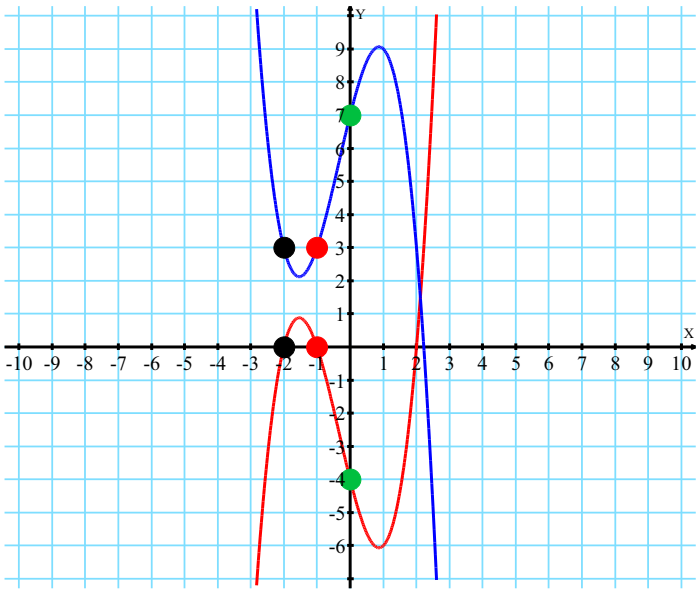
$y = 1/x^2$
$y = [1/(x - 2)^2] + 3$
$y = -1[[1/(x - 2)^2] + 3]$
$y = -1/(x - 2)^2 - 3$

Reflect

Reflect

15.

Vertical Reflection, Up 3 Units of $y = (x+1)(x+2)(x-2)$
To Get $y = -(x+1)(x+2)(x-2) + 3$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(x, - y + 3)
(-2, 0)	(-2, 3)
(-1,0)	(-1, 3)
(0, -4)	(0, 7)

Vertical Reflection, Then Shift Up 3 Units

Multiply y by -1, then add 3 to y.

$y = (x+1)(x+2)(x-2)$

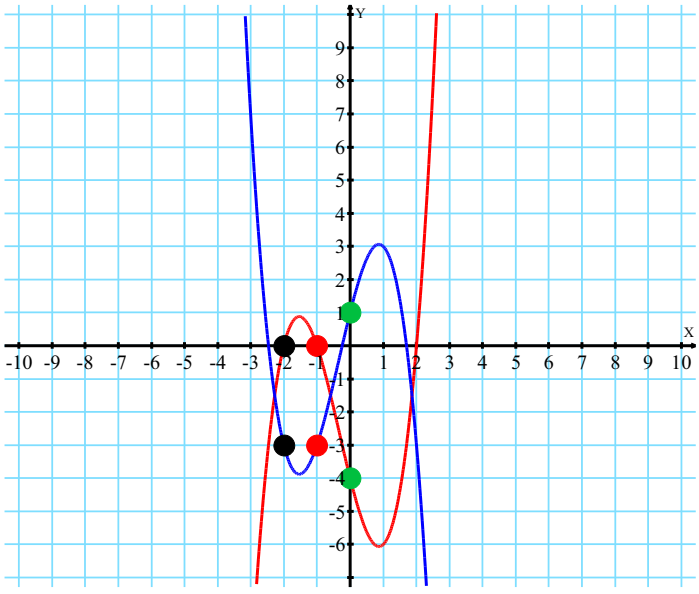
$y = -1[(x+1)(x+2)(x-2)] + 3$

$y = -(x+1)(x+2)(x-2) + 3$

Reflect → Reflect

16.

Up 3 Units, Then a Vertical Reflection of $y = (x+1)(x+2)(x-2)$
To Get $y = -[(x+1)(x+2)(x-2) + 3]$



Original Point (x, y)	Transformed Point (x', y')
(x, y)	(x, - (y + 3))
(-2, 0)	(-2, -3)
(-1,0)	(-1, -3)
(0, -4)	(0, 1)

Shift Up 3 Units, Then a Vertical Reflectiion

Add 3 to y, Then Multiply y by -1.

$y = (x+1)(x+2)(x-2)$

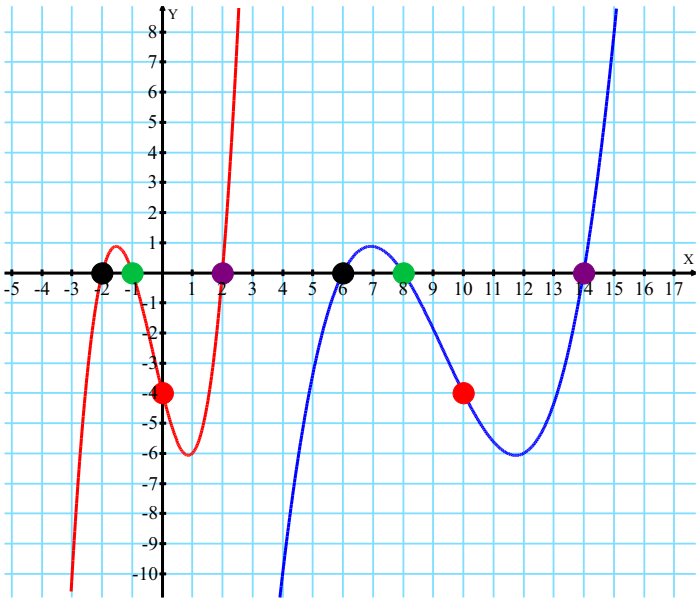
$y = -1[(x+1)(x+2)(x-2) + 3]$

$y = -(x+1)(x+2)(x-2) - 3$

Reflect → Reflect

17.

Horizontal Shift Right 5 Units, Then Horizontal Stretch by a factor of 2 of $y = (x+2)(x+1)(x-2)$ To Get $y = ((1/2)(x-6))((1/2)(x-8))((1/2)(x-14))$



Original Point (x, y)	Transformed Point (x', y')
(x, y) →	(2(x + 5) , y)
(-2, 0)	(6, 0)
(-1,0)	(8, 0)
(0, -4)	(10, -4)
(2, 0)	(14, 0)

Horizontal Shift Right 5 Units, Then Horizontal Stretch by 2.

Subtract 5 from x, then multiply x by the reciprocal of 2 which is 1/2. Finally, put in a(x-h) form.

$y = (x+2)(x+1)(x-2)$

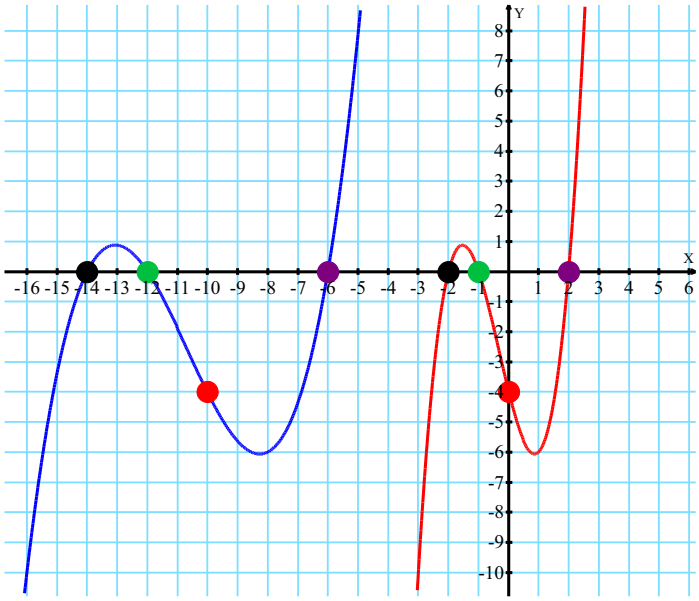
$y = ((1/2)x+2-5)((1/2)x+1-5)((1/2)x-2-5)$

$y = ((1/2)x-3)) ((1/2)x-4)) ((1/2)x-7))$

$y = ((1/2)(x-6))((1/2)(x-8))((1/2)(x-14))$

18.

Horizontal Shift Left 5 Units, Then Stretch by a factor of 2, of $y = (x+2)(x+1)(x-2)$ To Get $y = (1/2(x+14)) (1/2(x+12)) (1/2(x+6))$



Original Point (x, y)	Transformed Point (x', y')
(x, y) →	((2)(x -5), y)
(-2, 0)	(-14, 0)
(-1, 0)	(-12, 0)
(0, -4)	(-10, -4)
(2, 0)	(-6, 0)

Horizontal Compression by 1/2, Then Horizontal Shift Left by 5.

Add 5 to x, then multiply x by the reciprocal of 2 which is 1/2. Finally, put in a(x-h) form.

$y = (x+2)(x+1)(x-2)$

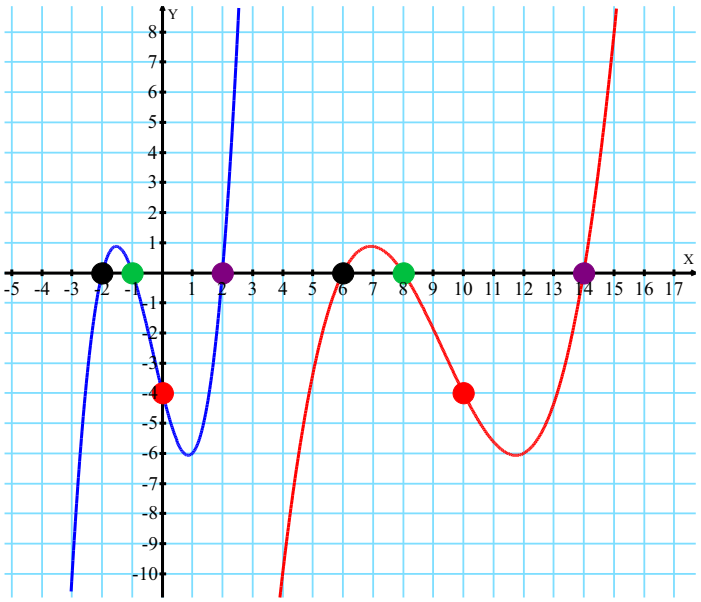
$y = (1/2x+2 +5)(1/2x +1 +5)(1/2x-2 +5)$

$y = (1/2x+7)(1/2x+6)(1/2x+3)$

$y = (1/2(x+14))(1/2(x+12))(1/2(x+6)$

19.

Horizontal Compression (Horizontal Shrink) by 1/2, Then Shift Left 5 Units of $y = ((1/2)x-4)((1/2)x-3)((1/2)x-7)$ To Get $y = (x+1)(x+2)(x-2)$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	((1/2)(x) -5 , y)
(6, 0)		(-2, 0)
(8, 0)		(-1, 0)
(10, -4)		(0, -4)
(14, 0)		(2, 0)

→ || ← ←

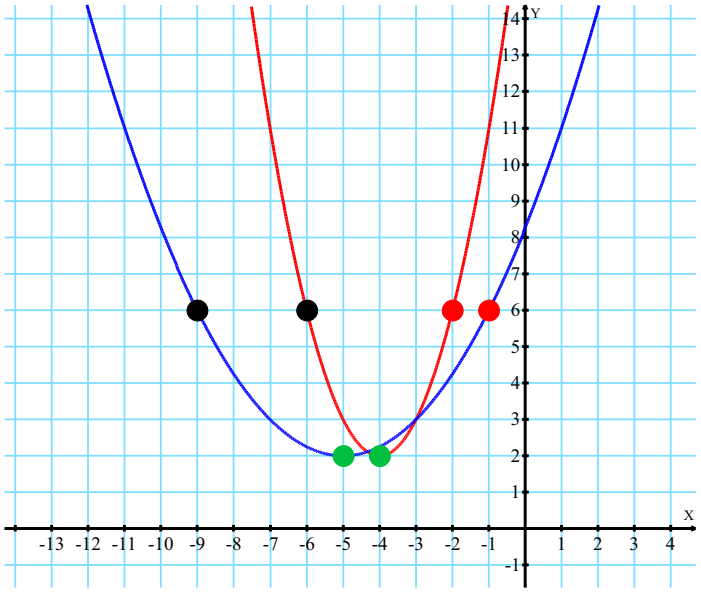
Horizontal Compression (Horizontal Shrink) by 1/2, Then Shift Left 5 Units

Multiply x by 2 which is the reciprocal of 1/2, then put into a(x-h) form, finally add 5 to x.

$y = ((1/2)x-4)((1/2)x-3)((1/2)x-7)$
$y = (2(1/2)x-4)(2(1/2)x-3)((2(1/2)x-7)$
$y = (x-4)(x-3)(x-7)$
$y = (x-4+5)(x-3+5)((x-7+5)$
$y = (x+1)(x+2)(x-2)$

20.

Horizontal Expansion (Stretch) by 2, Then Shift Right 3 Units of $y = (x+4)^2 + 2$ To Get $y = [(1/2)(x+5)]^2 + 2$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	(2 (x) +3 , y)
(-6, 6)		(-9, 6)
(-4, 2)		(-5, 2)
(-2, 6)		(-1, 6)

← || → ←

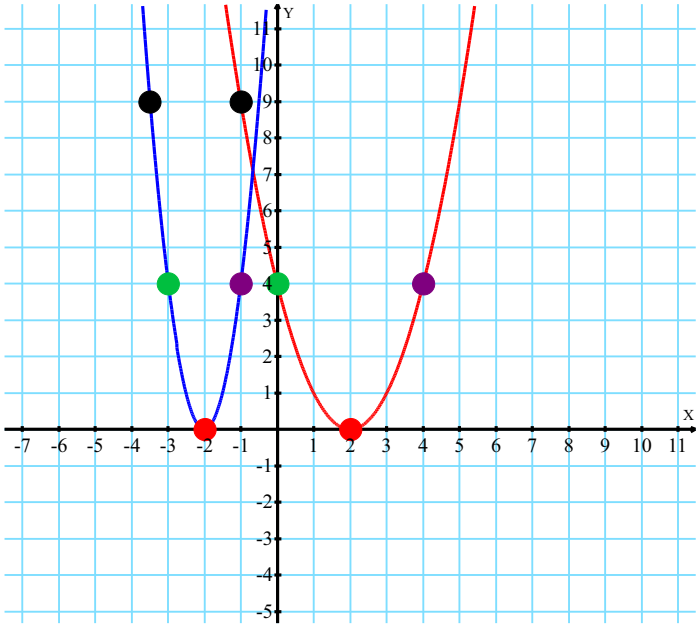
Horizontal Expansion (Stretch) by 2, Then Shift Right 3 Units

Multiply x by 1/2 which is the reciprocal of 2, then put in a(x-h) form (factor out the 1/2). Now subtract 3 from x.

$y = (x+4)^2 + 2$
$y = [(1/2)x+4]^2 + 2$
$y = [(1/2)(x+8 -3)]^2 + 2$
$y = [(1/2)(x+5)]^2 + 2$

21.

Horizontal Compression (Horizontal Shrink) by 1/2, Then Shift Left 3 Units of $y = (x-2)^2$ To Get $y = 4(x+2)^2$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	((1/2)(x) -3 , y)
(-1, 9)		(-3.5, 9)
(0, 4)		(-3, 4)
(2, 0)		(-2, 0)
(4, 4)		(-1, 4)

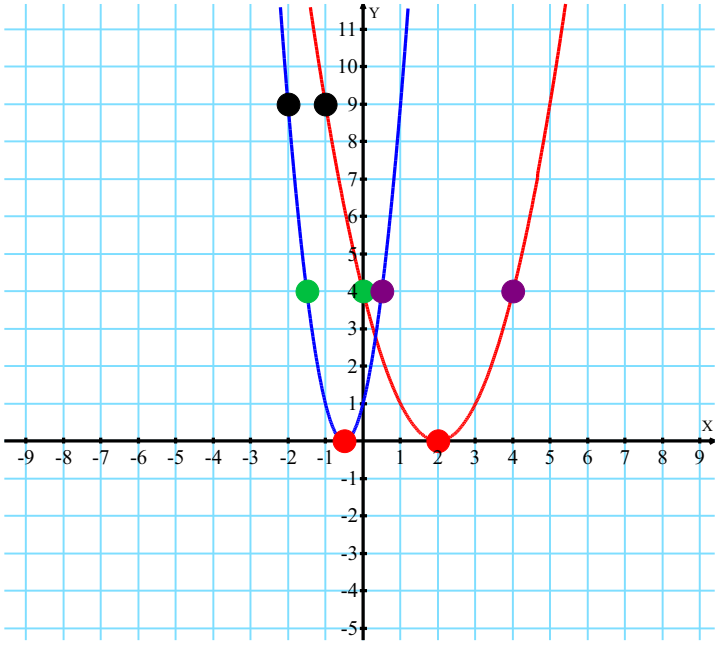
Horizontal Compression (Horizontal Shrink) by 1/2, Then Shift Left 3 Units

Multiply x by 2 which is the reciprocal of 1/2, then put into a(x-h) form. Next add 3 to x and simplify again.

$y = (x-2)^2$
$y = (2x-2)^2$
$y = (2(x-1))^2$
$y = (2(x-1 +3))^2$
$y = (2(x+2))^2$
$y = 4(x+2)^2$

22.

Shift Left 3 Units, Then Horizontal Compression (Horizontal Shrink) by 1/2 of $y = (x-2)^2$ To Get $y = 4(x+1/2)^2$



Original Point (x, y)		Transformed Point (x', y')
(x, y)	→	((1/2) (x -3) , y)
(-1, 9)		(-2, 9)
(2, 0)		(-1/2, 0)
(2, 0)		(-2, 0)
(4, 4)		(-1, 4)

Shift Left 3 Units, Then Horizontal Compression (Horizontal Shrink) by 1/2

Add 3 to x and simplify, then multiply x by 2 which is the reciprocal of 1/2. Next put it int a(x-h) form by factoring out the 2, and simplify again.

$y = (x-2)^2$
$y = (x-2 +3)^2$
$y = (x+1)^2$
$y = (2x+1))^2$
$y = (2(x+1/2))^2$
$y = 4(x+1/2)^2$